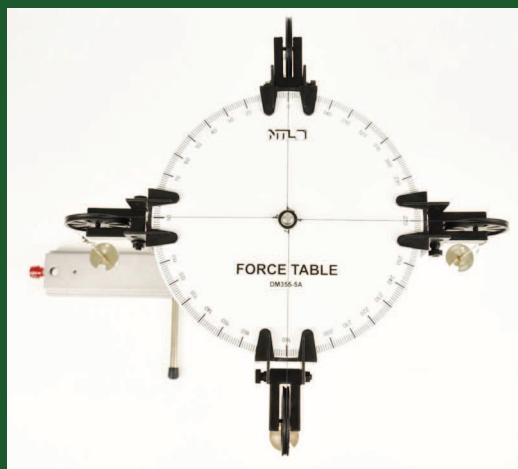


Student Experiments

Manual

FORCES AND TORQUE

P9160-1P



www.ntl.at

INDEX

1. EQUILIBRIUM ON THE HORIZONTAL FORCE TABLE

MEK 1.1	Composition of several forces
MEK 1.2	Force direction and application point

2. EQUILIBRIUM ON THE HORIZONTAL FORCE TABLE

MEK 2.0	Setup of the experiments
MEK 2.1	Equilibrium of torques
MEK 2.2	Force direction on a two-sided lever

3. COMPUTER- ASSISTED DATA LOGGING

MEK 3.0	Setup of the electronic data acquisition
MEK 3.1	Rotation - measuring of the moment of inertia
MEK 3.2	Dependence on the moment of inertia of the mass distribution
MEK 3.3	Rotation – moment of inertia

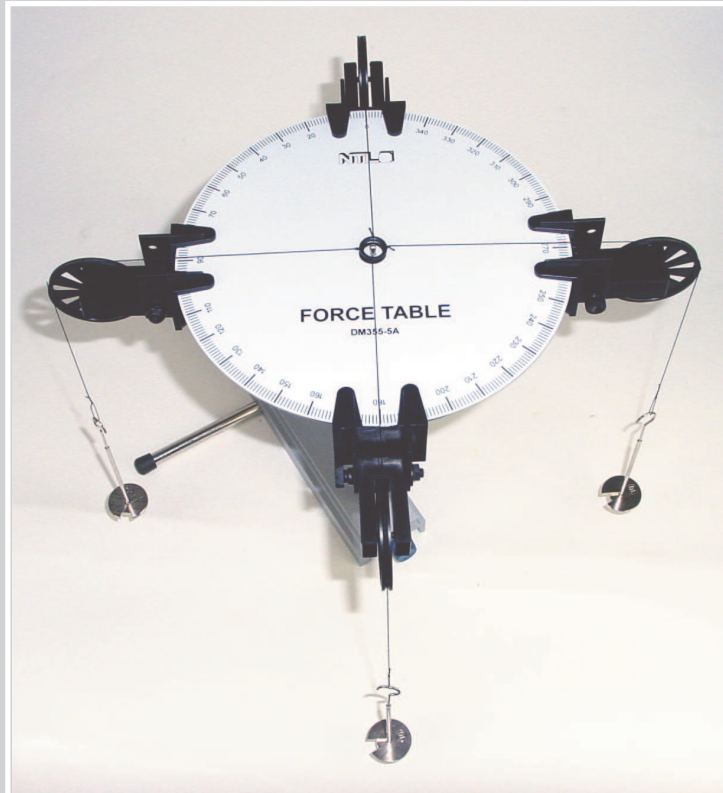
COMPOSITION OF SEVERAL FORCES

MEK 1.1

Required Kit:

P9901-4A Rail stand material

P9902-4P Forces and torque



Material:

- 1x Standrail, 300 mm
- 2x Rod 250 x 10 mm
- 2x End-cap for rods
- 1x Round bosshead
- 1x Force table
- 1x Ring with 4 threads for force table
- 4x Pulley with very low friction
- 4x Holder for slotted weights, 10 g
- 4x Slotted weight, 10 g



COMPOSITION OF SEVERAL FORCES

MEK 1.1

Basically, quantities in physics are either scalar or vector quantities. Scalars are quantities that have only a magnitude. Examples are for instance mass or energy.

Vector quantities have both magnitude and direction. An example: We must know how fast a body is and also the direction in which the body moves. Hence, velocity and acceleration are both vector quantities.

Speaking about forces, not only the magnitude is an important factor, but also the force direction. Hence, forces are vectors, too.

It depends on the dimension how many numbers you need to determine the vector exactly.

In the following experiments we are going to work at a force table. The experiment is two-dimensional and we will need two components to determine a vector exactly. The two components are either the magnitude and the reference angle (polar coordinates) or its Cartesian coordinates.

If more forces act on a body, we must add up the vectors to maintain the total force.

This can be done graphically by drawing the first vector, and then placing the tail of the second vector at the tip of the first arrow, then the tail of the third at the tip of the second, and so on. If you connect the tail of the first vector to the tip of the last vector you get the overall effect of these vectors – the resultant.

If only two different forces are given, we can draw a so called parallelogram of forces. The four sides represent the vectors of the acting forces and the diagonal is the vector of the resultant force.

N.B.:

In this case we prepare the experiments by making a graphical addition. If you need exact mathematical results, it is certainly possible to do the calculations for the experiments by using the sine and cosine functions to add the polar coordinates.

Preparation:

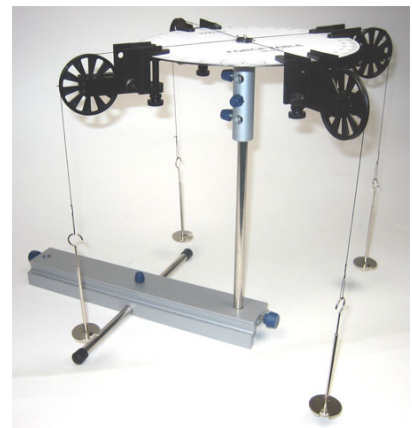
Setup as shown in picture.

Push one of the support rods, $L=250$ mm, through the cross hole of the stand rail and fix it with a knurled screw. Place the plastic caps at both ends of the support rod. Connect the second support rod, $L=250$ mm, to the force table by using the round bosshead. Fix the other end of the support rod vertically in the stand rail. At first, place 3 pulleys along the circumference of the force table in a user-defined position. Flap them down completely.

Screw the retaining dowel into the centre of the force table. Put the support ring with the four straps on it. Pass three of the four straps over the pulleys. Hook one holder for slotted weights onto each strap.

In the following experiments we work with three or four acting forces, and we always want to achieve equilibrium of forces.

The force consists of weights which act on the support ring over cords which lead over the pulleys.

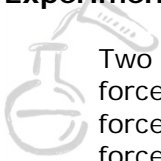


COMPOSITION OF SEVERAL FORCES

MEK 1.1

Equilibrium means that the forces are balanced, thus the resultant force is the null vector. If the forces are balanced, the support ring will float over the force table. A body remains at rest unless acted on by a resultant force. The retaining dowel makes sure that the support ring is not accelerated into the direction of the force if the acting forces are not in a state of equilibrium.

Experiment 1:



Two forces that have to be adjusted at the force table are given in the table. The third force should be adjusted so that the vector sum of the forces is zero. Thus, the third force must act in the opposite direction compared to the resultant of the first two forces; the magnitude must be equal. First, we determine the force by scale drawing, then we adjust the force at the force table and finally we check if the support ring does not move.

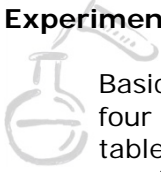


N.B.: Adjust the forces in the following way: Push the pulley (the elevation of the pulley support is used as a labeling) to the given angle φ and hook the given weights F to the holder for slotted weights.

The length of the force vectors corresponds to the magnitude of the force (the longer the vector of the force, the bigger the force). Mind using a reasonable scale in your graphical calculation since the drawing can result too small (for example your first step: 1 cm equals 0.1 N). The weight of the holder for slotted weights equals 0.1 N. (Please hold the support ring while adjusting the equilibrium; in that way it cannot slip over the dowel).

φ_1 [°]	F_1 [N]	φ_2 [°]	F_2 [N]	φ_3 [°]	F_3 [N]
0	1	120	1	240	1
0	1	120	0,3	197	0,9
0	0,8	90	0,6	217	1
0	0,8	60	0,8	210	1,4
0	1	30	1	195	1,9

Experiment 2:



Basically, we are going to repeat the first experiment. This time, however, we use all four pulleys. That means: The angle and the magnitude of three forces are given in the table. The fourth force should be determined in order to get a balance of the body. First we determine the fourth force by scale drawing. For this purpose the three forces are added up graphically (reasonable scale). The fourth force has the same magnitude as the resultant of the first three forces; the direction is opposite to the resultant. Please check the results at the force table.

$\phi 1 [^\circ]$	F1 [N]	$\phi 2 [^\circ]$	F2 [N]	$\phi 3 [^\circ]$	F3 [N]	$\phi 4 [^\circ]$	F4 [N]
0	100	90	100	180	100	270	100
0	100	45	80	180	100	225	80
0	80	60	60	197	90	225	35
0	100	80	60	110	60	238	145
0	120	130	60	185	60	240	45



Conclusion:

Physical quantities which are vectors have both magnitude and direction. If the forces acting on a body are in equilibrium, the body reacts as if no force was acting on it. The body does not change its state of motion because the resultant force is a zero vector.

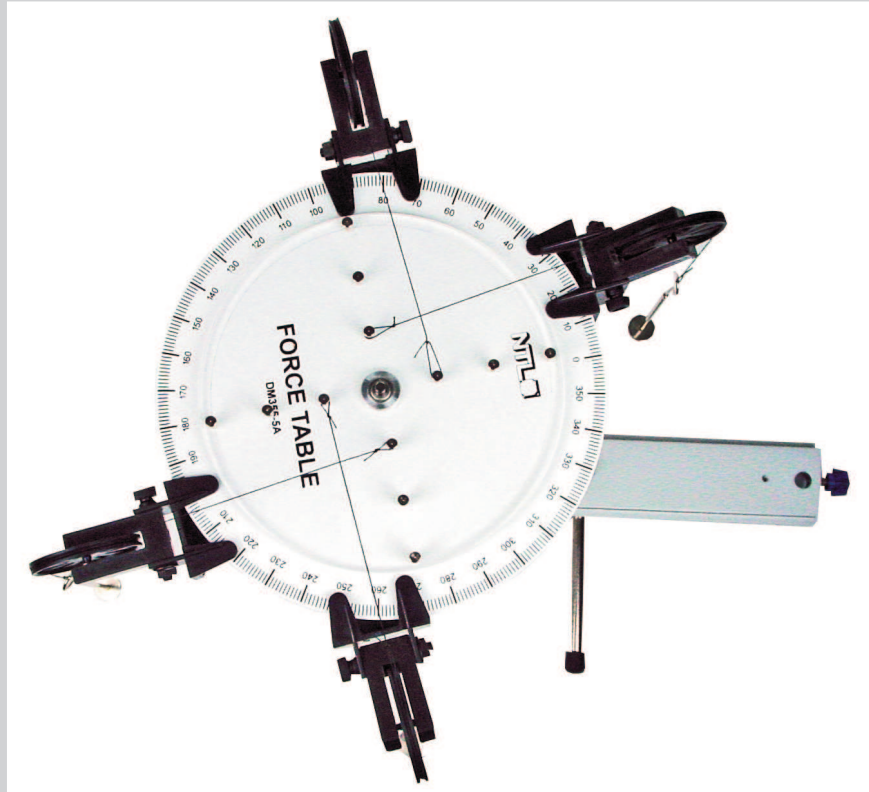
FORCE DIRECTION AND APPLICATION POINT

MEK 1.2

Required Kit:

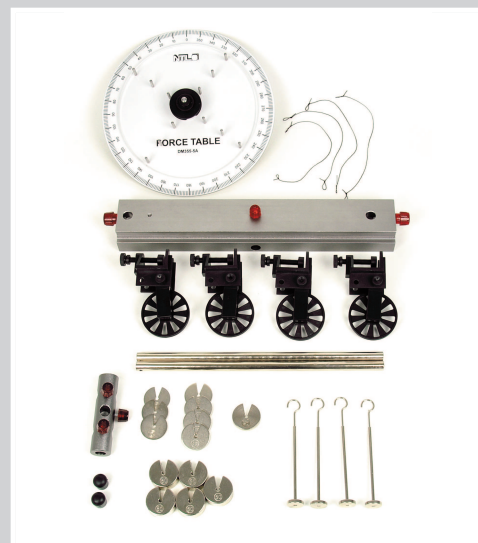
P9901-4A Rail stand material

P9902-4P Forces and torque



Material:

- 1x Standrail, 300 mm
- 2x Rod 250 x 10 mm
- 2x End-cap for rods
- 1x Round bosshead
- 1x Force table
- 1x Torque accessory for force table
- 4x Threads with loops for torque accessory
- 4x Pulley with very low friction
- 4x Holder for slotted weights, 10 g
- 3x Slotted weight, 5 g
- 4x Slotted weight, 10 g
- 1x Slotted weight, 20 g
- 5x Slotted weight, 50 g



FORCE DIRECTION AND APPLICATION POINT

MEK 1.2

Basically, quantities in physics are either scalar or vector quantities. There are three important factors: force direction, magnitude of the force and the point where the force acts on a body. If the force acts at the centre of mass of a body, the body can only be translated in one direction. If the same force has another application point, for instance at the edge of a wheel, it depends on the force direction if the body is only translated or if it rotates. The body will generally start to rotate if the prolongation of the vector of the acting force does not touch the centre of mass (respectively the fulcrum).

The longer the door handle, the easier it is to push the door handle down. The longer the lever, the easier it is to lift something up with the lever. Conclusion: Not only the magnitude and the direction of the force are decisive, but also the application point. Hence, a force can put a body into rotation. The torque of the force determines whether the body is going to move and in what way. Torque \vec{N} is a vector. In three dimensions it can be calculated with this equation:

$$\vec{N} = \vec{F} \times \vec{r}$$

In this case, F is the acting force and r is the vector between the centre of rotation and the application point of the force. In three dimensions N is perpendicular to F and r . If F and r are normal to each other, the magnitude of \vec{N} will be the biggest.

In our two-dimensional experiments there are only two possible directions for the torque. It acts either clockwise or anticlockwise. By convention, a clockwise torque is negative. The magnitude of the torque can be calculated with:

$$|\vec{N}| = |\vec{F}| * |\vec{r}| * \sin \theta$$

θ is the angle between the vectors F and r .

The relation torque-rotation is equal to the relation force-translation (linear motion). Therefore, there is a law of inertia also for rotation:

A body remains at rest if there is no acting torque. If the body is already rotating, the rotation will stay uniform.

(That is also true if the torques acting on a body are completely balanced.)

Our next experiment deals with bodies that have torques in equilibrium.

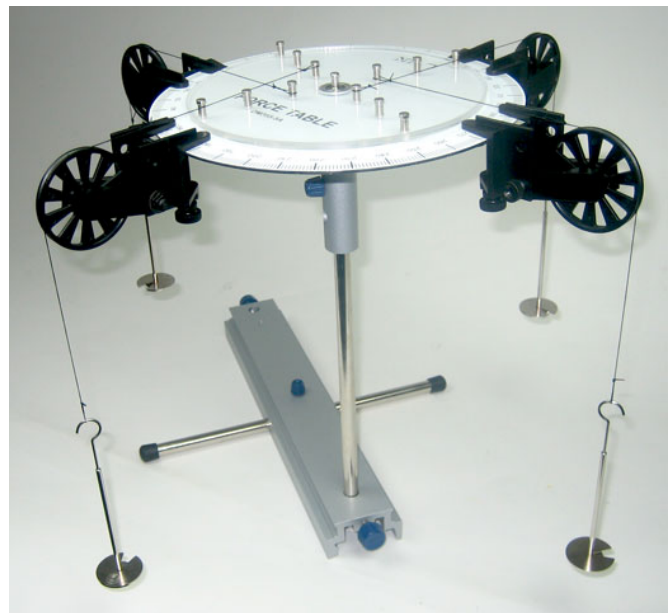
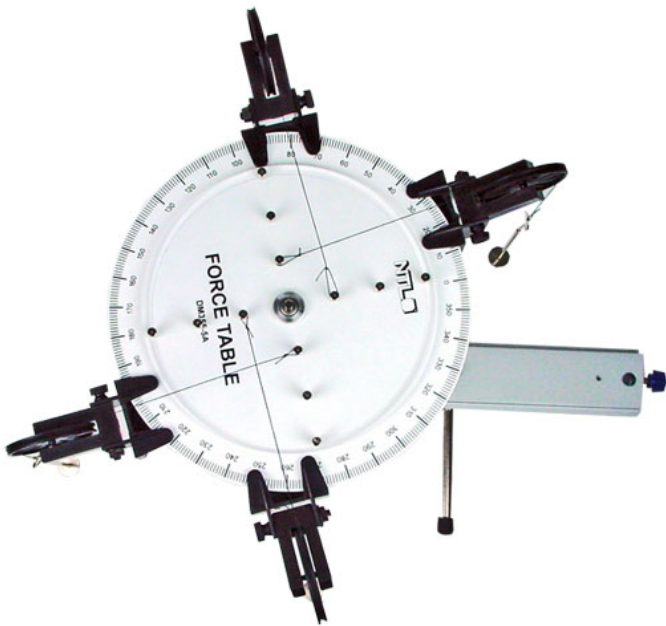
FORCE DIRECTION AND APPLICATION POINT

MEK 1.2

Preparation:

Setup as shown in picture.

Push one of the support rods, $L=250$ mm, through the cross hole of the stand rail and fix it with a knurled screw. Place the plastic caps at both ends of the support rod. Connect the second support rod, $L=250$ mm, to the force table by using the round bosshead. Fix the other end of the support rod vertically in the stand rail. Screw the torque accessory to the force table. Attach the four cords to the four inner dowels of the torque accessory (distance to centre: 2.5 cm; distance between dowels: 2.5 cm respectively). Adjust to the other ends of all the cords a holder for slotted weights. Fix the four pulleys so that there is a 90° angle between the lever arm (tie line between centre and application point) and its corresponding force. Fix the cords in order to accomplish equilibrium of torque. That means: two torques of equal size act in both directions (see picture).

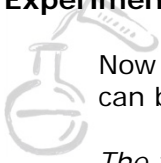


“Basic position”: torques are balanced.

FORCE DIRECTION AND APPLICATION POINT

MEK 1.2

Experiment 1:



Now load one of the holder for slotted weights with an additional weight of 20 g. What can be noticed?

The turntable starts to turn according to the acting torque.

Which holders must be loaded with a total mass of 20 g to accomplish a state of torque equilibrium?

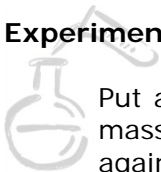
The 20 g can be randomly split to the two holders that act in opposite direction.

Which of the torques act in the same direction?

The two opposite to each other, respectively.

Remove the masses. Only the holders for slotted weights remain attached to the cords.

Experiment 2:



Put an additional weight of 50 g on the three holders for slotted weights. Increase the mass on one holder to a total mass of 120 g (holder 10 g + masses 110 g). We have again a disturbed equilibrium. To which retaining dowel must we attach one of the two counteracting cords to balance again the total torque? (before changing the cords it is important to position the pulleys in a way that the cord passes straight over the pulley. Check also the angle between the lever arm and the force. It must be again 90° because also the sine of this angle is decisive for the magnitude.

One of the two counteracting cords should be attached to the retaining dowel that can be found 2.5 cm further outside.

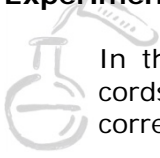
Is it possible to balance the torque by doubling two forces acting in the same direction without doubling also the counterforce? How?

You could attach both counteracting cords to the correspondent retaining dowels that are 5 cm away from the centre.

FORCE DIRECTION AND APPLICATION POINT

MEK 1.2

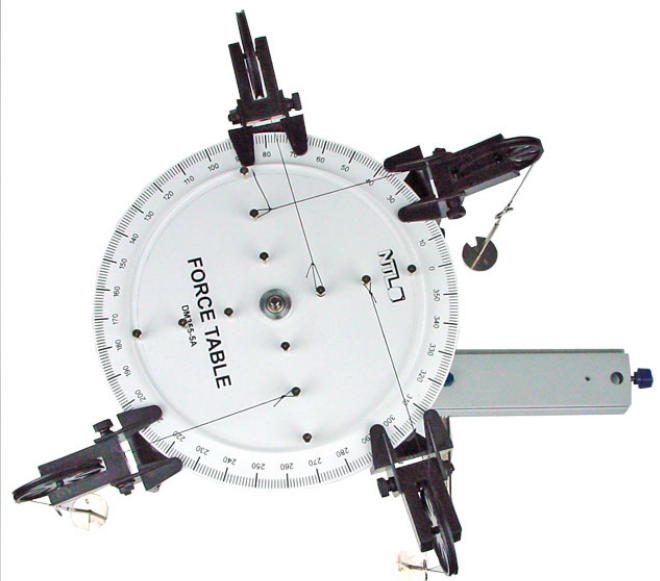
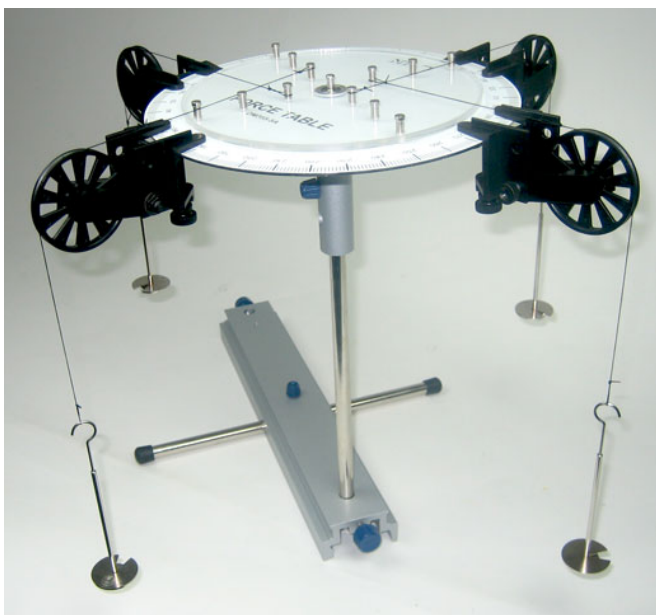
Experiment 3:



In this experiment three of the four torques act in the same direction. Attach three cords to the retaining dowels that are 5 cm away from the centre. Place the corresponding pulleys in the right position (see picture on the next page).

Only the holders for slotted weights and an additional weight of 5 g respectively are attached to the three cords. The fourth force must act on one of the four inner retaining dowels. This fourth torque is to balance the other three. What mass must be attached to the cord?

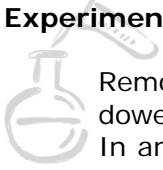
Attach a total mass of 90 g to the fourth cord so that the torque is balanced.



FORCE DIRECTION AND APPLICATION POINT

MEK 1.2

Experiment 4:



Remove one of the pulleys. Put three of the four cords around three inner retaining dowels (we do not need the fourth cord). Attach a weight of 0.6 N to one of the cords. In an angle of 90° attach a weight of 0.6 N. Attach a weight of 0.6 N also to the other two cords. What angle of lever arm and force is necessary to balance the torque (the angles must be equal; if they are not, there will be an infinite number of different solutions)? Find the solutions and set the result on the force table by using a set square. In that way you can test the equilibrium.

The angles must be 30° or 150° and 210° or 330° , respectively. The sine of the first two angles is 0.5, the sine of the second angles is -0.5.



Conclusion:

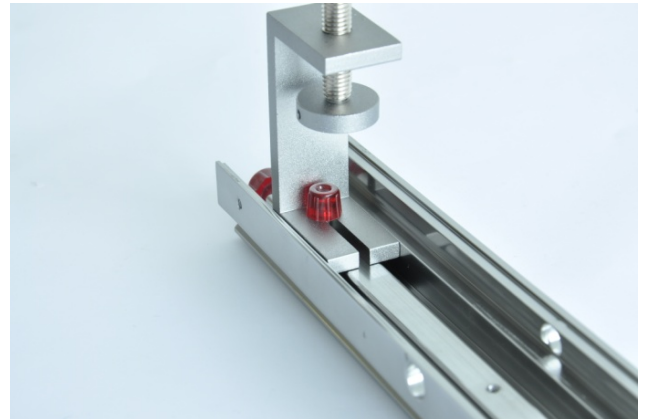
A torque in consequence of a force depends on the distance between the line of application of the force and the centre of rotation. (= lever arm)

If the distance between application point and centre of rotation = r , and if the angle between force direction and tie line application point – centre of rotation = α , the lever arm = $r \cdot \sin \alpha$.

The longer the lever arm, the bigger the torque (given a constant force).

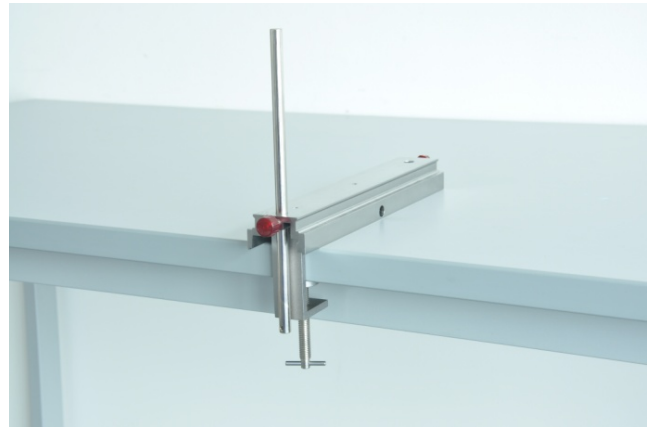
Law of inertia for rotation: A body remains in uniform rotation (constant angular velocity) or at rest or if there is no acting torque.

Push the table clamp in the intended guidance rail at the lower part of the stand rail, 30 cm, until it touches the clamp screw. Fix the table clamp with that screw.



Afterwards, attach the stand rail, 30 cm, at the table by using the table clamp. The drill hole for the stand rail must protrude the table.

Now insert the support rod, round, $L=250$ mm, into the drill hole of the stand rail.



N.B.: The height of fall should be at least 80 cm. Therefore the upper part of the support rod should be at least 90 cm above the floor.

Now put the round bosshead on the support rod, round, $L=250$ mm, and fix it.

Screw the acrylic disk onto the force table, put the shaft wheel onto the acrylic disk and fix it with a fixation screw.



SETUP OF THE EXPERIMENTS

MEK 2.0

Then put the assembled force table on the round bosshead and fix it with the screw.

Now fasten the guide pulley to the edge of the force table. The distance to the table should be as big as possible.



N.B. The ultrasonic sensor, which is used in the experiment, measures distances from 0.2 up to 6 m in an angle of 18°. If we want to measure the distance to an object (in our case the holder for slotted weights) and the object is too close to another object (in our case the table) the ultrasonic sensor could wrongly determine the distance to that object.

Cut a high tensile strength cord with a length of approximately 1 m from the 30 m roll.

N.B. The length of the cord should be the distance between floor and the height of the drive disks plus maximal an additional part of 15 cm. It should not be too short neither in order to fully utilize the total distance of fall (initial height until approximately 18 cm above the ultrasonic sensor).

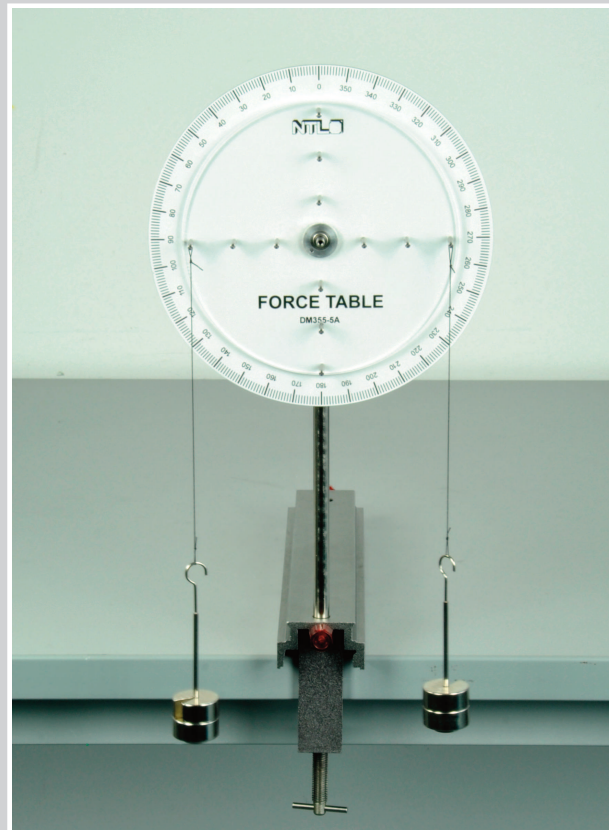
Make a knot at one end of the cord and make a strap at the other end. Attach the holder for slotted weights in the strap, pass the cord over the pulley and through the slot of the middle drive pulley ($r = 0.8 \text{ cm}$). The upper end of the guide pulley should be at an equal height to the drive pulleys in use.



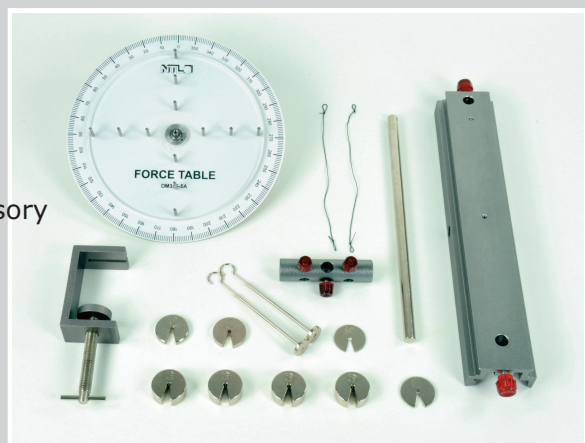
Required Kit:

P9901-4A Rail stand material

P9902-4P Forces and torque

**Material:**

- 1x Standrail, 300 mm
- 1x Rod 250 x 10 mm
- 1x Table clamp
- 1x Universal bosshead
- 1x Force table
- 1x Torque accessory for force table
- 2x Threads with loops for torque accessory
- 2x Holder for slotted weights, 10 g
- 1x Slotted weight, 5 g
- 1x Slotted weight, 10 g
- 2x Slotted weight, 20 g
- 4x Slotted weight, 50 g



The target of the experiment is to keep the torque accessory in balance, even if there are diverse distances from the pivot point.

Setup:

Arrangement according to the illustration.

The table clamp is fixed on the table in such way, that the drilling extends over the table edge. The support rod 250 mm is stuck into the drilling and fixed by means of the knurled screw. The force table is fixed vertical at the end of support rod by means of the round bosshead.

The two threads with loops for torque accessory are hung on the two outer pins. A holder for slotted weights inclusive slotted weights is hung on each thread.

In this experiment the target is always to keep the force table in balance. The forces are acting in form of weights on the pins of the torque accessory. Balance means that the forces have to cancel each other. The resulting force has to be zero. If the torque accessory is in balance, the pins are standing horizontal because if there is no force acting on a body he won't change his motion state.

Experiment 1:



At first slotted weights with 100 g are plugged on two of the opposite outer pins of the torque accessory. We can see that the torque accessory is kept in balance.

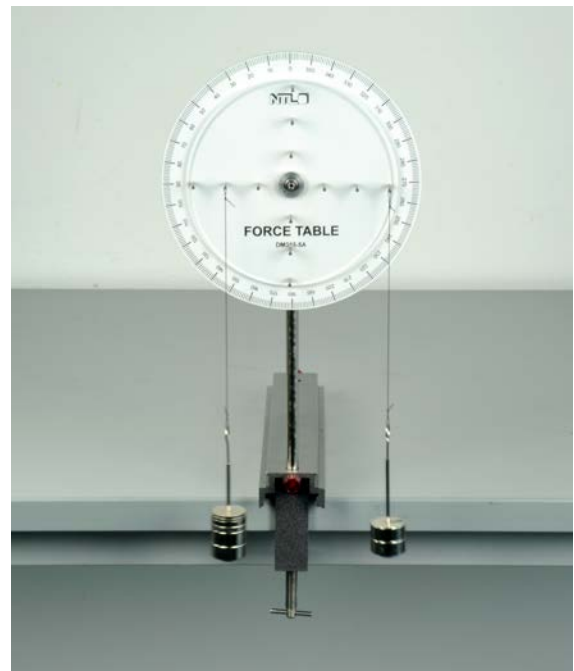
If we increase the weight on one thread by 10 g we can recognize that the torque accessory is not in balance any more.

Experiment 2:



The added 10 g are removed again. One loop is placed from the outer pin to the pin in the middle. The torque accessory is now in imbalance. How much weight do we have to put on the holder for slotted weights which is hanging on the middle pin to restore the balance?

The outer pin is located 7,5 cm away from the pivot. The middle pin is located 5,0 cm from the pivot. The mass on the outer pin is 110 g (inclusive the weight of the holder for slotted weights). The resulting force has to be zero.



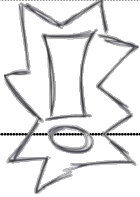
Thus results:

$$7,5 \times 110 = 5,0 \times f$$

Formed to "f":

$$7,5 \times 110 / 5,0 = 165 \text{ g}$$

Thus a mass of 155 g has to be put on the holder for slotted weights on the middle pin, to keep the torque accessory in balance.



Note:

The moment of inertia can be determined by means of a Dynamometer (2 N). (unbalanced load)



Conclusion:

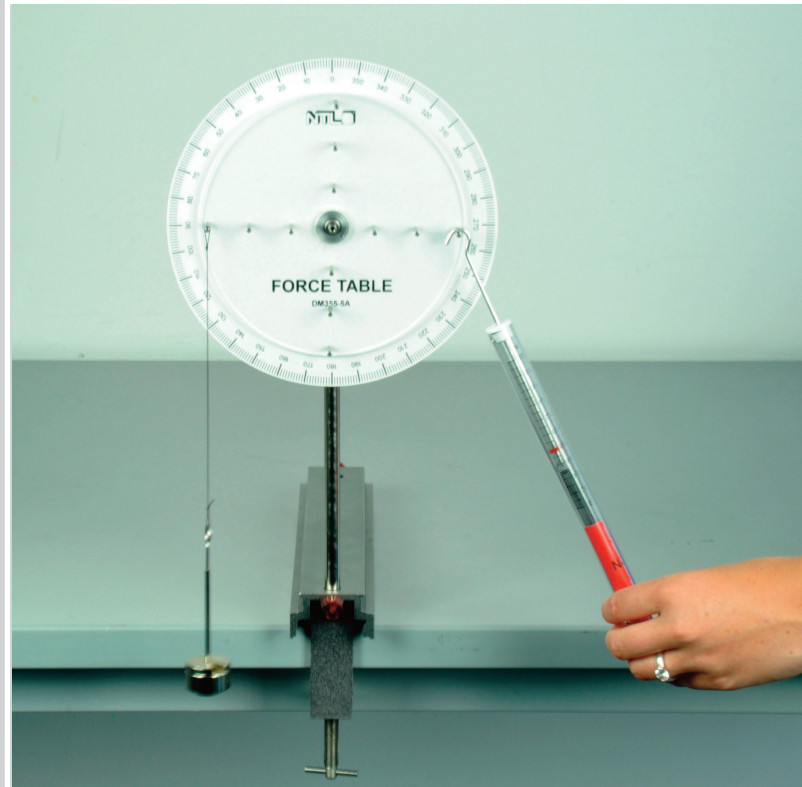
For the torque load the rectangular distance from the acting line of the force to the pivot is significantly. The torque accessory is in balance if all of the acting forces are cancelling each other. The sum of all forces (resulting force) has to be zero.

FORCE DIRECTION ON A TWO SIDED LEVER

MEK 2.2

Required Kit:

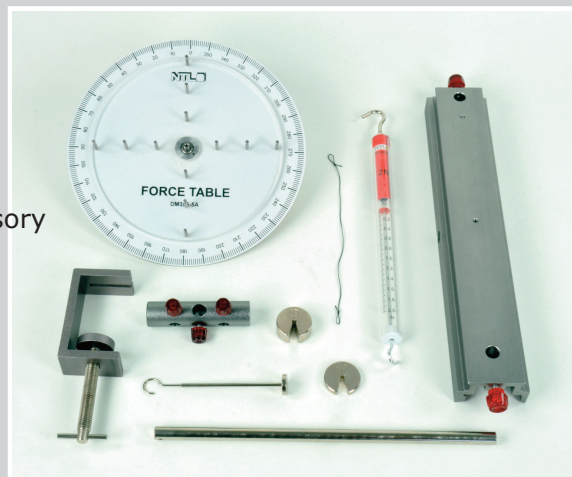
P9901-4A Rail stand material
P9901-4B Mechanics 1
P9902-4P Forces and torque



Material:

1x Standrail, 300 mm
1x Rod 250 x 10 mm
1x Table clamp
1x Universal bosshead
1x Force table
1x Torque accessory for force table
2x Threads with loops for torque accessory
1x Holder for slotted weights, 10 g
1x Slotted weight, 20 g
1x Slotted weight, 50 g

Additionally required:
1x Dynamometer 2 N



FORCE DIRECTION OF A TWO – SIDED LEVER

MEK 2.2

The dependence of the force angle to the pivot is to be investigated.

Setup:

Arrangement according to the illustration.

The table clamp is fixed on the table in such way, that the drilling extends over the table edge. The support rod 250 mm is stuck into the drilling and fixed by means of the knurled screw. The force table is fixed vertical at the end of support rod by means of the round bosshead.

One thread with loop for torque accessory is hung on the outer pin of the torque accessory and a holder for slotted weights is fixed on the other side of the thread. A slotted weight with a mass of 50 g and one slotted weight with a mass of 20 g are plugged on the holder for slotted weights. The torque accessory is aligned in such way that "180°" on the scale are exactly in the plumb line of the weight. The dynamometer is fixed on the opposite outer pin.

Experiment 1:



It is to be investigated at which angle of the dynamometer to the pivot is at least or at most force needed to get the pin in a horizontal position. (90 ° on the scale)

The holder for slotted weights is now brought in a horizontal position by means of the dynamometer. At first the dynamometer is aligned vertical. The rectangular distance to the pivot is now 7,5 cm. A force of about 0,8 N is to be seen on the dynamometer. So this is exactly the weight force of the slotted weights.

FORCE DIRECTION OF A TWO – SIDED LEVER

MEK 2.2

Experiment 2:



Next it is tried to turn the dynamometer by 30° outward. The pin on which the weight is hanging should even stand on 90°. Which force can be read on the dynamometer?

The rectangular line on the acting line of the force (moment arm) is:

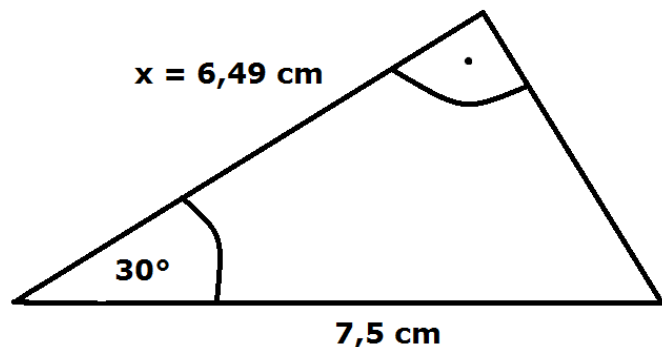
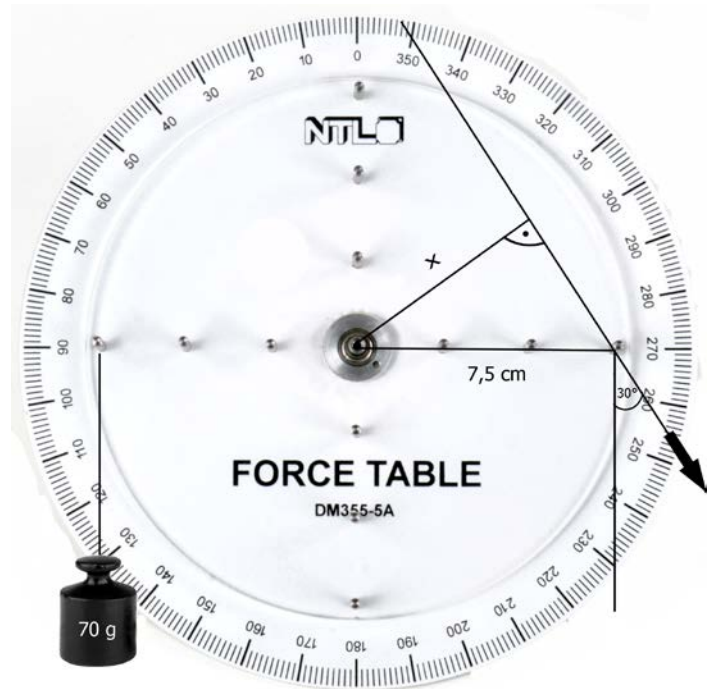
$$7,5 \times \cos(30) = 6,49 \text{ cm}$$

To get the forces back in balance the dynamometer has to absorb the force of the weight:

$$7,5 \times 70 = 6,49 \times f$$

$$\text{Formed to „f“: } 7,5 \times 70 / 6,49 = 80,83 \text{ g}$$

So the dynamometer has to show a force of about 81 g at its scale.



Erkenntnis:

Entscheidend für das Drehmoment einer Kraft ist nicht der Angriffspunkt der Kraft von der Drehachse, sondern der Normalabstand der Wirkungsline von der Drehachse.

SETUP OF THE ELECTRONIC DATA ACQUISITION

MEK 3.0

Plug the ultrasonic sensor in the interface.

Connect the interface transformer to the interface and the power supply.

Connect the interface with the USB port of a computer (Coach 6 must be installed). If required turn on the interface.

Open Coach 6, go to the menu bar, click on "File" and select "New".

In the dialogue window "Activity Options" go to "Activity Type" and click "Measurement". Select the panel from the drop-down list: "Sample Interface".

Click on "Display" in the menu bar. Go to "Window Layouts" and select "2x2".

Go to the toolbar and click on the stopwatch ("Measurement Settings"). Go to "Method" and "Type". Choose "Time-based" in the drop-down list, fill in the "Measuring time" (fill in the value which is indicated in the experiment) and adjust the "Frequency" to 20 per second. Then click "OK".

Right-click in the first empty application window. Select "Display Diagram". A dialogue window opens. Select CH(SAMPLE): Ultrasonic Motion Detector.

Next step: Right-click once more in the application window with the diagram already in display and select "Edit a Diagram". "Column": Click on "C2" and change "Max" to "1.1".

N.B. The maximum value should be bigger than the height of fall (there are reference values for a height of fall of 80 cm).

Confirm the entry by a click on "OK".

Next step: Right-click in the second empty application window and choose "Display Table". A dialogue window opens. Choose again ultrasonic motion detector and click "OK".

The first two application windows are used for data acquisition. The current height "C2" is shown in the diagram with reference to the time already elapsed "C1" (distance-time diagram). The longer the measurement, the smaller the height. Therefore we will receive negative values for velocity and acceleration when we analyse the diagram. In further calculations we do not need the negative sign.

Right-click in the application window "Table" and choose "Edit a Table". A dialogue window opens and you can edit the columns.

Optional: Right-click in the third empty application window and choose "Display Text". Choose a title for the application window and click "OK". The window can be used to take down notes.

In the fourth application window we create another diagram. We will use this window to work on the data.

SETUP OF THE ELECTRONIC DATA ACQUISITION

MEK 3.0

Right-click in the fourth empty application window and select "Display Diagram". The dialogue window "Select a Diagram" opens. Click on the button "New Diagram". The dialogue window "Create/Edit a Diagram" opens. First, select a name for the diagram, e.g. "Analysis". Click on "C1", go to "Connection" and select "Clock" in the drop-down list. New options are shown in the dialogue window. At this stage we do not change them. Then choose "C2", go to "Connection" and select CH(SAMPLE) ultrasonic motion detector. Insert "s(t)" for "Quantity". The "Max" is the same as in the first diagram. Then click on "C3". Go to "Connection". This time select "Formula". New options are shown in the dialogue window. In the field axis select "First vertical". Click on the button "Edit formula". Since we want to display the current velocity in "C3", we have to insert for formula: "Derivative([s(t)])". Confirm the entry by a click on "OK".

N.B. The negative sign is used to compensate the other negative sign which we mentioned before. It is not really necessary.

Insert for "Quantity" " $-v(t)$ " and for "Unit" "m/s". Choose for "Max" "1.0". Go to "Column" and click on "C4". Again select in the drop-down list "Formula". C4 is going to display the current acceleration. Therefore insert in the field formula "DerivativeSecond([s(t)])". Insert for "Quantity" " $-a(t)$ " and for "Unit" "m/s²". In the "Axis" section choose "Second vertical" and insert for "Max" "1.1". The "Max" for the single columns have to be changed in the analyzing process. Then click "OK".

N.B. If you do the experiment "moment of inertia qualitative", you need the fourth application window only if you want an exact analysis (and not only a qualitative one).

Position the ultrasonic sensor under the holder for slotted weights. The holder should not be in contact with the sensor since it would fall on it in each experiment. The ultrasonic sensor should not change position afterwards.

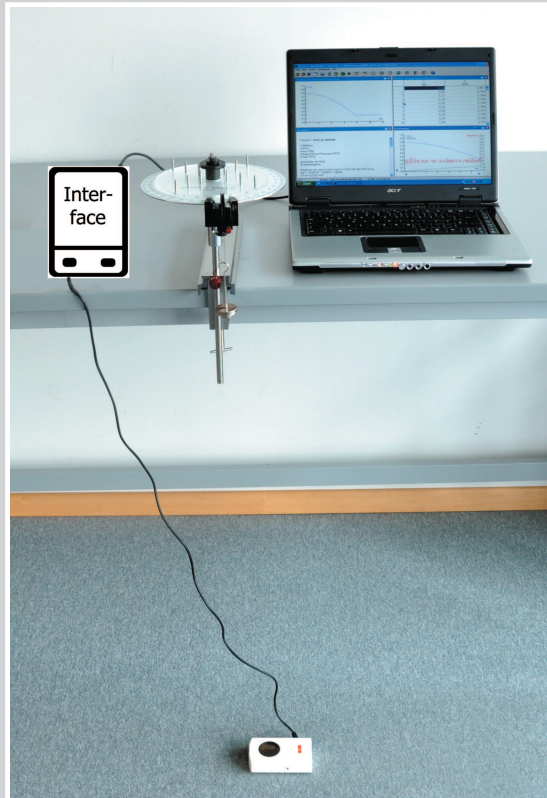
ROTATION - MEASURING OF THE MOMENT OF INERTIA

MEK 3.1

Required Kit:

P9901-4A Rail stand material

P9902-4P Forces and torque

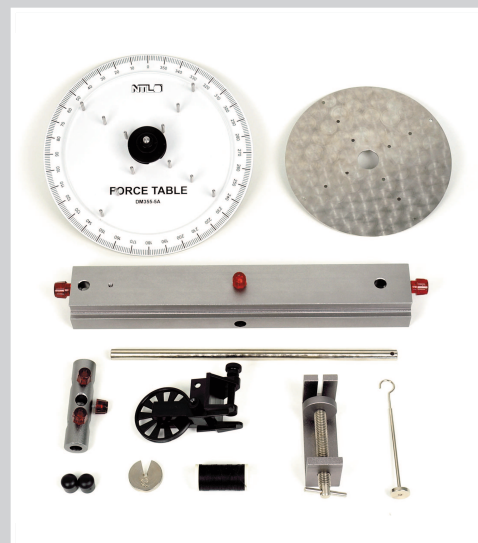


Material:

- 1x Standrail, 300 mm
- 1x Table clamp
- 1x Rod 250 x 10 mm
- 1x Round bosshead
- 1x Force table
- 1x Torque accessory for force table
- 1x Pulley with very low friction
- 1x Thread, very strong, (roll of 30 m)
- 1x Holder for slotted weights, 10 g
- 1x Slotted weight, 20 g
- 1x Additional mass for Torque accessory

Additionally required:

- 1x Interface
- 1x Motionsensor



ROTATION – MEASURING OF THE MOMENT OF INERTIA

MEK 3.1

Linear motion: A certain force is applied on a body and the acceleration is inversely proportional to the mass of the body.

Rotation: The angular acceleration depends on the acting force, the application point of the force and the moment of inertia. The moment of inertia is the rotational analog to mass of the linear motion and (as the linear motion) acts in the opposite direction of the accelerating force.

In the following experiment we are going to determine the moment of inertia of two disks with different masses. We use a distance-time function and calculate the moment of inertia of the disks. Additionally, we calculate the moment of inertia also theoretically and then we compare the results.

Theory:

Equation of motion:

Newton's second law states that the rate of change of motion depends on the applied force (acceleration, slowing-down, change of direction). In linear motion, the required force is directly proportional to the acceleration. The proportional factor of the force and the acceleration is the mass of the body.

$$\vec{F} = m * \vec{a} \quad (1)$$

Force \vec{F} and acceleration \vec{a} are vectors, mass m is a scalar. In a first step we deal with one-dimensional motions and therefore we do not need to calculate with vectors.

The acceleration is the second derivative of the distance-time function $s(t)$.

$$\frac{d^2 s(t)}{dt^2} = a(t) \quad (2)$$

Given a uniform accelerated motion, the acting force and therefore also the acceleration do not change in time. To prove that, we have to integrate equation (2) twice. So we find the following relation:

$$s(t) = \frac{a * t^2}{2} + v_0 * t + s_0 \quad (3)$$

In this equation v_0 is the initial velocity of the body, s_0 is the distance already travelled by the body at the beginning of measuring ($t = 0$). If we know v_0 , s_0 , $s(t)$ and t we can determine the acceleration by transforming equation (3). If we also know the mass or the force we can determine the other quantities by doing a derivative of equation (1).

If we talk about rotations the relations are similar. There is a rotational analog to each of the quantities described above by translation motion (linear displacement). In translation, every point of a figure moves by the same amount. This is not the case if we talk about rotation. The points on the inner part of a rotating disk travel a much shorter distance than the points at the edge of the disk. However, the points sweep over the same angle. The travelled distance is related to the travelled angle φ . We can calculate the distance that a point on the rotating disk (distance r to the center) travels with the following equation:

$$s = r * \varphi \quad (4)$$

ROTATION – MEASURING OF THE MOMENT OF INERTIA

MEK 3.1

Note that the dimension of the angle is in radian and not in degrees. Furthermore we can specify an angular velocity ω and an angular acceleration α (in general, φ , ω and α are also vector quantities).

$$\omega = \frac{d\varphi}{dt}, \alpha = \frac{d\omega}{dt} \quad (5,6)$$

$$v = r * \omega, a = r * \alpha \quad (7,8)$$

In the rotation, the equivalent to force is the torque. Torque \vec{N} is a vector and is calculated by the vector product of the acting force \vec{F} and the vector between the center of rotation and the application point of the force \vec{r} .

$$\vec{N} = \vec{F} \times \vec{r} \quad (9)$$

In the following experiments \vec{F} and \vec{r} are perpendicular and have only one component which is not 0 (namely F and r). Therefore \vec{N} also has only one component which is not 0, too. We will call this component N_z . Now we can simplify equation (9) to:

$$N_z = F * r \quad (10)$$

As mentioned before, the equivalent to mass is the moment of inertia I. Generally, the moment of inertia is a symmetric tensor of 2nd degree and has 9 components in three dimensions. 6 of them are independent. It can be written as a 3x3 matrix. This matrix can be transformed so that only the figures in the main diagonal are not zero. They are equivalent to the moments of inertia if the body rotates around the main axis. The numbers in the second diagonal are responsible for the change of the rotation axis. This happens only if the body does not rotate around the main axis. We use the following equation to describe rotation:

$$\vec{N} = I \vec{\alpha} \quad (11)$$



N.B.: The operation in equation (11) is a contraction of tensor I with the vector $\vec{\alpha}$.

In the experiment the disks rotate only around the z-axis. Therefore we only need the moment of inertia for the rotation around the z-axis I_z (3x3 component of I). We simplify the equation above to:

$$N_z = I_z * \alpha \quad (12)$$

ROTATION – MEASURING OF THE MOMENT OF INERTIA

MEK 3.1

It is again a simple multiplication of two scalars I_z and α (z component of the angular acceleration vector). If we substitute equation (10) into equation (12) we get the following equation:

$$F * r = I_z * \alpha \quad (13)$$

In this equation there are only scalars. Therefore it can be easily transformed and we get:

$$I_z = \frac{F * r}{\alpha} \quad (14)$$

Theoretical calculation of the torque:

If we want to put a body in rotation, we must overcome the moment of inertia. The moment of inertia depends on the mass and the distribution of the mass around the rotation axis. Let us take for instance a rotating disk. The rotation axis goes through the center of the disk. A point of the disk with mass m_i and distance r_i to the center has the following influence on the total moment of inertia of the disk:

$$I_i = m_i * r_i^2 \quad (15)$$

The sum of all moments of inertia of every point of the disk is the total moment of inertia of the disk:

$$I = \sum_i I_i = \sum_i m_i * r_i^2 \quad (16)$$

If the mass is distributed equally, as it is the case with a real disk, the sum must be substituted by an integral:

$$I = \int r^2 dm \quad (17)$$

If we want to calculate the moment of inertia of a full disk (complete disk) or a full cylinder we get the following equation after several transformations and integration:

$$I = \frac{m_{\text{total}} r^2}{2} \quad (18)$$

m_{total} is the total mass of the disk (cylinder) and r is the radius of the disk. If we calculate a disk with a round hole in the center or the moment of inertia of a hollow cylinder we get the following result:

$$I = \frac{m_{\text{total}} * (r_i^2 + r_a^2)}{2} \quad (19)$$

In this equation m_{total} is again the total mass of the disk (of the cylinder), r_i is the radius of the hole of the disk (inner radius of the cylinder), r_a is the distance from the center to the edge (outer radius). In both equations a homogeneous mass distribution is necessary. One result of those equations is that a hollow cylinder with a total mass concentrated at the edge ($r_i = r_a$) has a moment of inertia which is twice as big as the moment of inertia of a full cylinder with the same radius and the same mass.

In this experiment we are going to measure the moment of inertia of the rotating part of the force table (once without the aluminum disk and once with the disk). First, we calculate both

ROTATION – MEASURING OF THE MOMENT OF INERTIA

MEK 3.1

moments of inertia theoretically. For that purpose we do a few simplifications. We consider the shaft wheel only as a cylinder with a homogeneous mass distribution. We consider the acrylic disk with the holding pins for slotted weights as a disk with homogeneous mass distribution.

Now we weigh the acrylic disk, the shaft wheel (without fixation screw) and the aluminum disk. Furthermore, we determine the radiuses of the cylinder and the acrylic disk as well as the inner and outer radius of the aluminum disk.

	m [kg]	r_a [m]	r_i [m]	I_z [kgm ²]
acrylic disk	0,1100	0,07885	-	0,0003420
cylinder	0,0140	0,02500	-	0,0000044
Aluminium disk	0,2021	0,07885	0,02500	0,0006914

Table 1: moments of inertia of the single rotating components theoretically calculated

In the first experiments we do not use the aluminum disk. We can calculate the moment of inertia of the rotating component I_p at the force table as the sum of the single moments of inertia (acrylic disk + cylinder).

$$I_{p,th} = (0,00034195 + 0,00000438) \text{ kgm}^2 = 0,0003464 \text{ kgm}^2 = 0,3464 \text{ gm}^2$$

In the following experiments we will also use the aluminum disk. The total moment of inertia is then the sum of all three moments of inertia (Table 1):

$$I_{A,th} = 0,0010378 \text{ kgm}^2 = 1,0378 \text{ gm}^2$$

Preparation:

Setup see MEK 2.0 Setup of the experiments

Setup of the electronic data acquisition:

See MEK 3.0 Setup of the electronic data acquisition



N.B. The measuring time in the single experiments is for a fall distance of approximately 60 cm. If the fall distance differs from this value, you must change the measuring time.

ROTATION – MEASURING OF THE MOMENT OF INERTIA

MEK 3.1

Experiment 1 – We determine the moment of inertia of the plexiglass disk with shaft wheel:



Measuring time: 5 s

We do the first experiment with an accelerating mass (holder for slotted weights + slotted weights) of 30 g. The holder for slotted weights weighs 10 g, therefore we add a slotted weight of 20 g. Wrap the cord with the accelerating mass around the central drive pulley ($r = 0.8 \text{ cm}$) until you reach the required height of fall. Hold the acrylic disk with a finger (there are reference values for a height of fall of $h = 0.8 \text{ m}$). You can see the current height (distance between holder for slotted weights and ultrasonic sensor) on the ULAB.

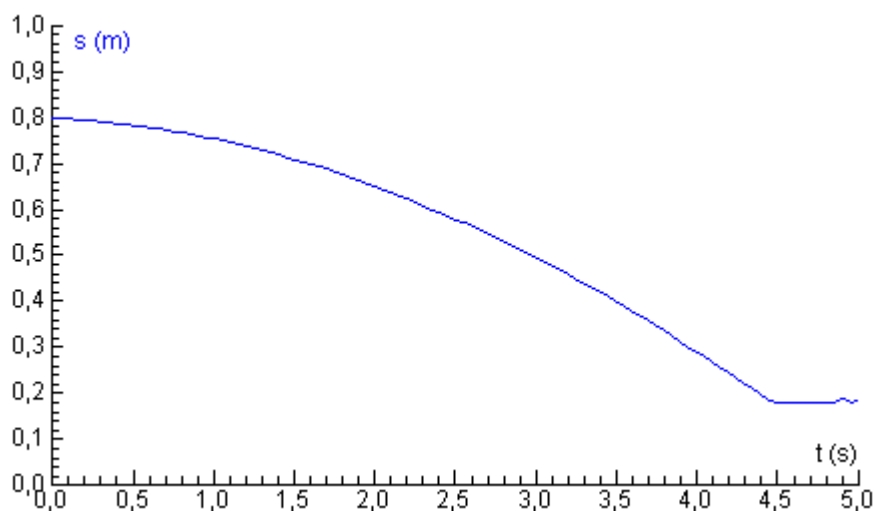
Please try to reduce possible interfering effects, as for instance vibrations or rotation of the holder for slotted weights, as much as possible before the start of the measurement.

Release the acrylic disk and simultaneously start the Coach data acquisition by clicking on "Start" in the menu bar.

Due to the gravitational force the holder for slotted weights with the slotted weight accelerates. The falling mass is responsible for a theoretically constant torque on the acrylic disk. The acrylic disk rotates with uniform acceleration.

The ultrasonic sensor measures the signals 20 times per second and transmits them to the ULAB and the computer where they are pictured in a distance-time diagram and we receive a distance-time function. During the fall the function should be equal to a semiparabola. The starting point is the maximum of the semiparabola. Coach 6 calculates the numeric derivative of the distance-time diagram. So we get the velocity-time diagram (1st derivative) and the acceleration-time diagram (2nd derivative).

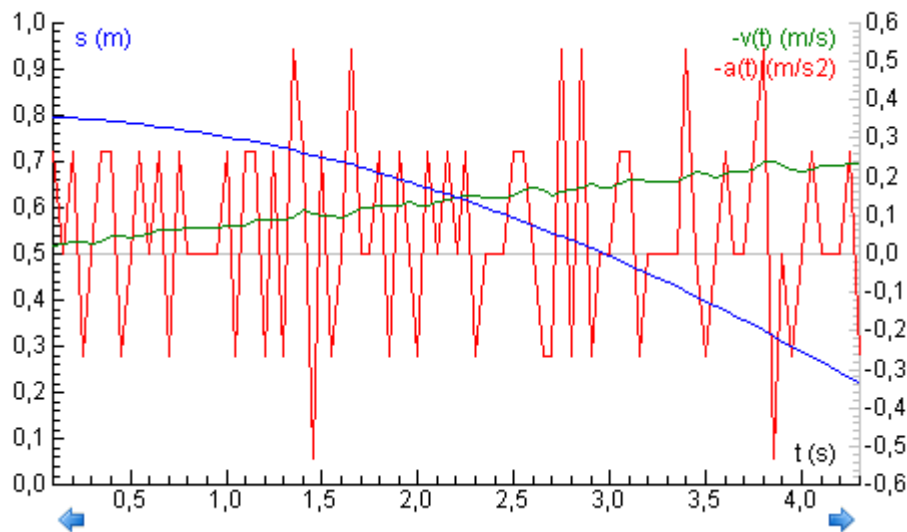
Catch the falling mass before the cord is completely unwrapped. You can do that by either stopping the acrylic disk or by catching the slotted weight. For our evaluation we only deal with the accelerating mass.



Graph 1: distance-time diagram of the falling mass

ROTATION – MEASURING OF THE MOMENT OF INERTIA

MEK 3.1



Graph 2: diagram to evaluate the motion



N.B. If there are big leaps (during the fall) in the distance-time diagram, there must be an error in the measurement (for instance the ultrasonic sensor did not measure the holder for slotted weights). Please repeat the measurement in that case. The diagram should be similar to the diagram in Graph 2.

In the diagram you can recognize the linear slope of the velocity. The acceleration should be constant, bigger variations are due to smaller interfering effects of the motion (e.g. friction). The bigger the accelerating mass, the smaller are the relative interfering effects. However, we use the distance-time function for the evaluation. That function can be described exactly by a square function. In order to receive a more precise curve for the acceleration, we have to smooth the curve in the derivative.

After a successful experiment you must save the data to keep them active. Otherwise the existing data will be replaced with the new data if you start the measurement again.

Evaluation:

In equation (8) the acceleration of the holder for slotted weights and the angular acceleration of the acrylic disk are related to each other. We can transform equation (8). The result is:

$$\alpha = \frac{a}{r} \quad (20)$$

Now we can substitute α into equation (14). The result is:

$$I_z = \frac{F * r^2}{a} \quad (21)$$

ROTATION – MEASURING OF THE MOMENT OF INERTIA

MEK 3.1

F is the force acting on the acrylic disk. The gravitational force F_G and the resulting force $-F$ from the moment of inertia of the acrylic disk act on the accelerating mass (holder for slotted weights and 20 g slotted weights). The resultant force F_{res} is the sum of these two forces.

$$F_{res} = F_G + (-F) = m_b * g - F \quad (22)$$

In this equation $m_b = 30 \text{ g}$ is the accelerating mass and $g = 9,81 \text{ m/s}^2$ is the acceleration due to gravity. The resultant force, however, is the outcome of:

$$F_{res} = m_b * a \quad (23)$$

Whereat a is the resultant acceleration. By putting the equations (22) and (23) in one equation and by rearranging it we get the following result:

$$F = m_b * (g - a) \quad (24)$$

Now we can substitute equation (24) into equation (21) and we get the equation for the moment of inertia:

$$I_z = \frac{m_b * r^2 * (g - a)}{a} = m_b * r^2 * \left(\frac{g}{a} - 1\right) \quad (25)$$

We know all the factors except the one for acceleration a . Now we need Coach 6 to evaluate the collected data and determine the acceleration.

The equation of motion for the falling mass is:

$$h(t) = \frac{a * t^2}{2} + v_0 * t + h_0 \quad (26)$$

The initial velocity v_0 should actually be 0 m/s and h_0 should be 0.8 m. This is not the case, however, since there are small interfering effects at the beginning of the fall. It would be possible to find three equations in three variables (a , v_0 , s_0) if we knew three relations between t and $h(t)$. These simultaneous equations could be solved. Because of the fact that we know much more of the relations between t and $h(t)$ (all measured points of data during the fall) we could create a much more accurate quadratic function through the points of data thanks to the computer. With this method we can determine the coefficients that are equal to $a/2$, v_0 , s_0 . Task: Approximate the quadratic function with the measured data.

Coach 6: Right-click in the diagram "Analysis" and first choose "Create/Edit a Diagram". Click on column "C1": choose "0.1" for "Min" and a little bit less than the moment of slowing-down

ROTATION – MEASURING OF THE MOMENT OF INERTIA

MEK 3.1

of the accelerating mass (for example "4.4" as shown in the diagram above) in "Max". So we can make sure to use only the data which are meaningful for the analysis. After the slowing-down, equation (23) does not describe the motion of the accelerating mass anymore.

Then insert the data for column "C3" and "C4". Adjust the "Min" and "Max" values in a way that the graphs $v(t)$ and $a(t)$ in the duration of the fall are completely visible in the diagram (do not choose minimum and maximum values that are too small or too big – see evaluation diagram). If the accelerating mass is for example 30 g, the evaluation should be between 0.1 s and 4.4 s (see figure 2). Confirm the inserted data by a click on "OK".

Now right-click again in the diagram Analysis go to "Process/Analyze" and select "Function Fit". The dialogue window "Function Fit" will open. Choose in "Column" " $s(t)$ " and in the drop-down list "Function Type" " $f(x)=ax^2+bx+c$ " (quadratic function). With this function we will approximate the points of data. Then click on "Estimate". The computer calculates four values: the three coefficients a , b , c of the quadratic function and the standard deviation. The three coefficients a , b and c mean: half of the acceleration a ($a/2$), initial velocity v_0 and distance travelled at the beginning of the measurement s_0 . The standard deviation tells us how much the single data points deviate from the calculated graph.

Write the three coefficients in the correspondent space in the first line of Table 1. We determine the moment of inertia from equation (25).

m_{inertia} [kg]	m_b [g]	r [m]	$a/2$ [m/s ²]	a [m/s ²]	v [m/s]	s [m]	I_z [kgm ²]
0,155	30	0,008	0,0260	0,0520	0,0240	0,8016	0,00036
0,165	40	0,008	0,0352	0,0704	0,0297	0,8020	0,00035
0,175	50	0,008	0,0431	0,0862	0,0372	0,8011	0,00036

Table 2: Moments of inertia experimentally determined

You will receive the values for the second and the third line by repeating the measurement with the corresponding accelerating masses.

Now we take the three determined values, add them up and divide the sum by three. The result is the mean value of the moment of inertia $I_{P,ex}$.

$$I_{P,ex} = 0,357 \text{ gm}^2$$

The theoretically calculated value I_{th} was:

$$I_{P,th} = 0,346 \text{ gm}^2$$

Now we can determine the relative deviation r_p of the value determined through the experiment in respect of the theoretical value:

$$r_p = \frac{I_{P,ex} - I_{P,th}}{I_{th}} \quad (27)$$

$$r_p = 0,032 = 3,2\%$$

We have a deviation of 3.2% between the two values.

ROTATION – MEASURING OF THE MOMENT OF INERTIA

MEK 3.1

Experiment 2 – Determination of the moment of inertia of the acrylic disk with shaft wheel and aluminium disk:



Measuring time: 9s

In order to carry out the second experiment, we have to make minor modifications in the setup of the experiment.

Remove the cord with the accelerating mass from the shaft wheel.

Unscrew the fixation screw of the shaft wheel and remove the shaft wheel from the acrylic disk.

In the next step we put the aluminum disk onto the acrylic disk.

Take a fixation screw and fix the shaft wheel again to the acrylic disk. Clamp the cord with the accelerating mass to the middle drive pulley ($r = 0.8 \text{ cm}$).

The adjustments in the Coach program, the procedure of the experiment and the evaluation are similar to the points in experiment 1.

We are going to measure the moment of inertia. For that purpose we use the accelerating masses taken down in Table 3, we determine the mean value and compare it to the value that we have already calculated theoretically.

m_{inertia} [kg]	m_b [kg]	r [m]	$a/2$ [m/s ²]	a [m/s ²]	v [m/s]	s [m]	I_z [kgm ²]
0,357	0,03	0,008	0,0091	0,0182	0,0097	0,8024	0,00103
0,367	0,04	0,008	0,0124	0,0248	0,0139	0,8030	0,00101
0,387	0,06	0,008	0,0186	0,0372	0,0172	0,8022	0,00101
0,407	0,08	0,008	0,0242	0,0484	0,0190	0,8020	0,00103

Table 3: Moments of inertia experimentally determined

The mean value of the moment of inertia is:

$$I_{A,\text{ex}} = 1,02 \text{ gm}^2$$

The theoretically calculated value I_{th} was:

$$I_{A,\text{th}} = 1,04 \text{ gm}^2$$

The relative deviation is:

$$r_A = -1,9\%$$



Conclusion:

The moment of inertia of a body tells us the resistance of this body being set in motion by a torque. The moment of inertia is the rotational analog to the quantity mass in translation motion. Since a body can have more than one rotating axes, the torque has several components (in three dimensions there are 9 components, 6 of them are free. You can transform it so that only three of those components are not 0. Those three components describe the rotation around the main axes of the body).

The moment of inertia of a body is directly proportional to its total mass and depends on the distribution of mass, which means it depends on the geometry of the body and its rotating axis. Consider that you have two bodies and one of them is twice as heavy as the other; both bodies should have the same geometry and should rotate around the same axis. Then the moment of inertia of the heavy body should be twice as big as the moment of inertia of the lighter body.

The unit of the moment of inertia is kgm^2 .

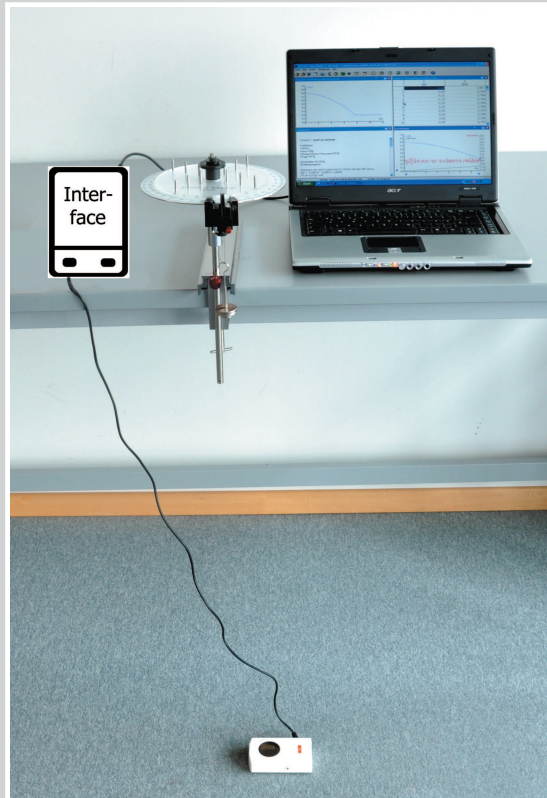
DEPENDENCE ON THE MOMENT OF INERTIA OF THE MASS DISTRIBUTION

MEK 3.2

Required Kit:

P9901-4A Rail stand material

P9902-4P Forces and torque



Material:

- 1x Standrail, 300 mm
- 1x Table clamp
- 1x Rod 250 x 10 mm
- 1x Round bosshead
- 1x Force table
- 1x Torque accessory for force table
- 1x Pulley with very low friction
- 1x Thread, very strong, (roll of 30 m)
- 1x Holder for slotted weights, 10 g
- 4x Slotted weight, 10 g
- 2x Slotted weight, 20 g
- 4x Slotted weight, 50 g
- 1x Additional mass for Torque accessory

Additionally required:

- 1x Interface
- 1x Motionsensor



DEPENDENCE ON THE MOMENT OF INERTIA OF THE MASS DISTRIBUTION

MEK 3.2

In the last experiment we have theoretically discussed in what way the moment of inertia depends on the distribution of masses. In the following experiment we want to discuss it from an experimental, qualitative point of view.

Preparation:

Setup see Chapter I. Set up of the experiments

Setup of the electronic data acquisition:

See Chapter II. Setup of the electronic data acquisition



N.B. The measuring time in the single experiments is for a fall distance of approximately 60 cm. If the fall distance differs from this value, you must change the measuring time.

Experiment 1:



Measuring time: 10 s

Attach to all of the inner holding pins of the acrylic disk two 20 g slotted weights and two 10 g slotted weights, respectively (we can use the 50 g slotted weights on the inner track only if the acrylic disk and the shaft wheel are disassembled).

The accelerating mass should be 40 g. Therefore we must put three 10 g slotted weights on the holder for slotted weights.

Wrap the cord with the accelerating mass around the middle drive pulley ($r = 0.8 \text{ cm}$) until you reach the required height of fall. Hold the acrylic disk with a finger. You can see the current height (distance between holder for slotted weights and ultrasonic sensor) on the ULAB.



N.B. In this experiment we discuss the effects on the moment of inertia only qualitatively. Therefore the value of the selected height of fall is not important. In order to be able to compare the four following experiments the height of fall should always be the same.

Please try to reduce possible interfering effects, as for instance vibrations or rotation of the holder for slotted weights, as much as possible before the start of the measurement.

Release the acrylic disk and simultaneously start the Coach data acquisition by clicking on "Start" in the menu bar.

DEPENDENCE ON THE MOMENT OF INERTIA OF THE MASS DISTRIBUTION

MEK 3.2

The holder for slotted weights is accelerated and puts the acrylic disk into rotation. The ultrasonic sensor measures the signals 20 times per second and transmits them to the ULAB and the computer where they are pictured in a distance-time diagram and we receive a distance-time function. During the fall the function should be equal to a semiparabola. The starting point is the maximum of the semiparabola.

Catch the falling mass before the cord is completely unwrapped. You can do that by either stopping the acrylic disk or by catching the slotted weight. For our evaluation we only deal with the accelerating mass.

If you do not want to cancel the recorded function, you must save the data. Right-click in the diagram and choose "Copy Column" and "s". In that way the recorded data will be saved in another column and is not replaced when you carry out a new measurement.

Experiment 2 – We determine the moment of inertia of the acrylic disk with shaft wheel and aluminum disk:



Measuring time: 10 s

Remove all the slotted weights from the inner holding pins and place a 50 g slotted weight on each of the middle holding pins, respectively. Wrap the cord up and repeat the experiment with the same accelerating mass.

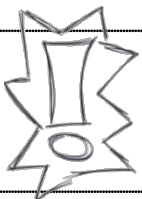
The distance-time function should be again a parabola. Copy the values as you did in the last experiment.

Experiment 3:



Measuring time: 10 s

Now place the four 50 g slotted weights on the outer holding pins of the acrylic disk and repeat the experiment. You can copy the values as you did in the last experiments.



N.B. *You must copy the data only if you want to record further functions in the same file and if you do not want to overwrite the old data.*

DEPENDENCE ON THE MOMENT OF INERTIA OF THE MASS DISTRIBUTION

MEK 3.2

Experiment 4:



Measuring time: 10 s

In this last experiment we replace the four 50 g slotted weights by the aluminium disk and repeat the experiment. (the aluminium disk has a mass of approximately 200 g ($m = 202,1$ g)). This can be compared to the total mass of the four 50 g slotted weights.

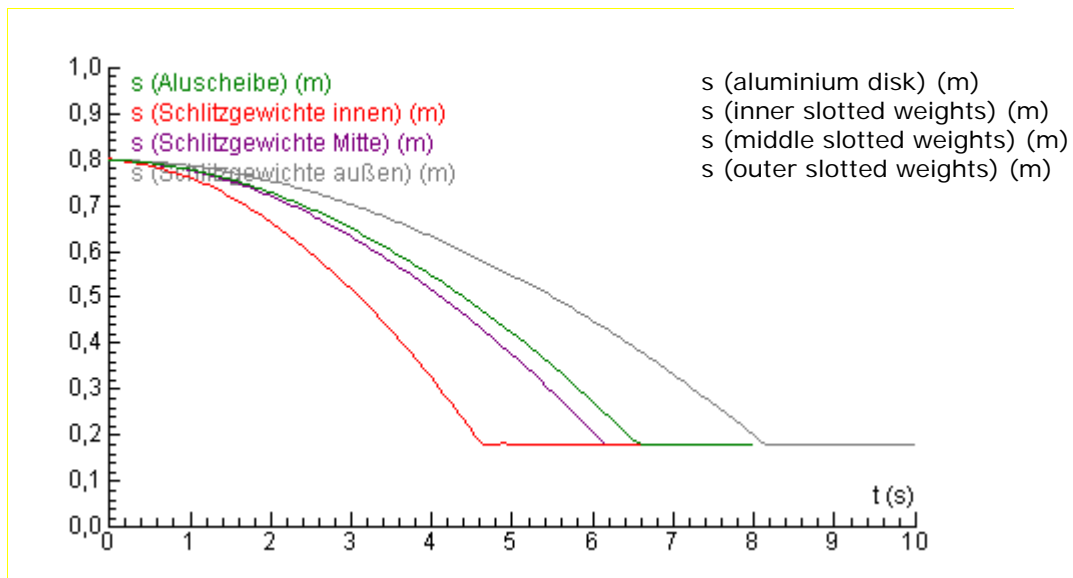


Figure 1: Distance-Time diagrams of the falling mass given a varying distribution of mass

Observation, comparison:

All of the four graphs are semiparabolas. The further the mass is away from the center, the wider the parabola, the longer the time of fall. The function for the homogeneous distribution of mass on the aluminium disk is the one most similar to the function with the slotted weights positioned at the middle holding pins. The longer time of fall is mainly due to the smaller acceleration (initial velocity and initial height should be equal or insignificantly different in all of the three experiments).

The accelerating torque is also equal in all of the three experiments. Conclusion: The moment of inertia must have become bigger since acceleration and torque are proportional to each other (the moment of inertia is the factor of proportionality). The moment of inertia has thus become bigger, although the total mass of the rotating body has not changed. The factor that has changed is the distribution of mass.

DEPENDENCE ON THE MOMENT OF INERTIA OF THE MASS DISTRIBUTION

MEK 3.2

Evaluation:

If we want a qualitative comparison of the moments of inertia but also a more precise evaluation, we need a fitting of the graphs of motion with quadratic functions. We can do that in a further Coach 6 diagram by substituting the acceleration into the following equation:

$$I_z = m_b * r^2 * \left(\frac{g}{a} - 1\right) \quad (1)$$

In the equation I_z is the moment of inertia, $m_b = 40 \text{ g}$ is the accelerating mass, $r = 0.008 \text{ m}$ is the radius of the middle drive pulley, $g = 9.81 \text{ m/s}^2$ is the acceleration due to gravity and a is the recorded acceleration. You can write the values in the table below.



N.B. The shown area of the x-axis (time axis) in the evaluation diagram has to be readjusted in each new evaluation. This is necessary because the fit considers all the data of the whole area, but in our case we do only need the data of the fall (Adjust the minimum value of the time axis to 0.1, so that the initial interferences are not considered. Adjust the maximum value of the time axis to the value where the fall ends).

	m_{total} [kg]	m_b [kg]	r [m]	$a/2$ [m/s ²]	a [m/s ²]	v [m/s]	s [m]	I_z [kgm ²]
inner slotted weights	0,365	0,04	0,008	0,0245	0,0490	0,0216	0,8033	0,00051
middle slotted weights	0,365	0,04	0,008	0,0142	0,0284	0,0150	0,8037	0,00088
outer slotted weights	0,365	0,04	0,008	0,0082	0,0164	0,0098	0,8032	0,00153
aluminium disk	0,367	0,04	0,008	0,0125	0,0250	0,0137	0,8028	0,00100

Table 1: Moments of inertia with a given different distribution of mass



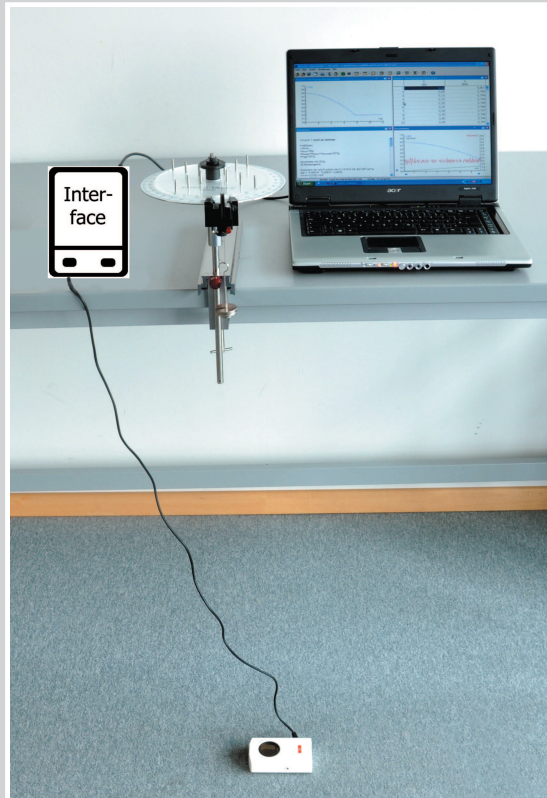
Conclusion:

The moment of inertia strongly depends on the geometry and the distribution of mass of the rotating body. The greater the distance between the rotating mass and the centre of rotation, the bigger the moment of inertia.

Required Kit:

P9901-4A Rail stand material

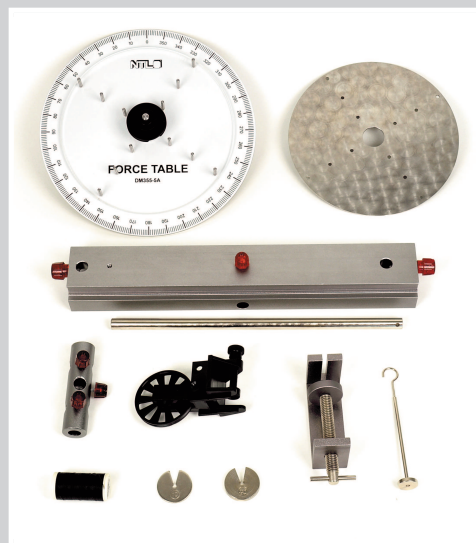
P9902-4P Forces and torque

**Material:**

- 1x Standrail, 300 mm
- 1x Table clamp
- 1x Rod 250 x 10 mm
- 1x Round bosshead
- 1x Force table
- 1x Torque accessory for force table
- 1x Pulley with very low friction
- 1x Thread, very strong, (roll of 30 m)
- 1x Holder for slotted weights, 10 g
- 1x Slotted weight, 10 g
- 1x Slotted weight, 20 g
- 1x Additional mass for Torque accessory

Additionally required:

- 1x Interface
- 1x Motionsensor



ROTATION – MOMENT OF INERTIA

MEK 3.3

In this experiment we want to examine how the torque depends on the radius (center to application point of force) and on the accelerating mass.

Theory:

$$\vec{N} = \vec{F} \times \vec{r} \quad (1)$$

In the experiment \vec{F} and \vec{r} are perpendicular. Since we only calculate the absolute values of the vectors (N,F,r), the equation above simplifies to:

$$N = F * r \quad (2)$$

As we discussed in the experiment next to last, we can calculate the acting force F in our experiment with the equation:

$$F = m_b * (g - a) \quad (3)$$

In this equation $m_b = 30 \text{ g}$ is the accelerating mass, $g = 9,81 \text{ m/s}^2$ is the acceleration due to gravity and a is the acceleration that we are going to measure in the following experiments. So we get the following result for the torque:

$$N = m_b * (g - a) * r \quad (4)$$

The torque can also be calculated with:

$$N = \frac{I * a}{r} \quad (5)$$

I is the moment of inertia of the rotating body. The equation can be transformed to:

$$a = \frac{N * r}{I} \quad (6)$$

Substitute a from equation (6) into equation (4), do the necessary transformations and then you find the following equation:

$$N = \frac{m_b * r * g}{1 + \frac{m_b * r^2}{I}} \quad (7)$$

In our experiments $m_b * r^2 / I$ is always much smaller than 1. In that case equation (7) simplifies to:

$$N = m_b * r * g \quad (8)$$

The torque is nearly proportional to the accelerating mass and to the radius.

Preparation:

Setup see Chapter I. Setup of the experiments

Setup of the electronic data acquisition:

See Chapter II. Setup of the electronic data acquisition



N.B. The measuring time in the single experiments is for a fall distance of approximately 60 cm. If the fall distance differs from this value, you must change the measuring time.

Experiment 1a:

Measuring time: 8 s

In the first experiment we want to measure the torque that acts on the rotating body (acrylic disk, also possible acrylic-aluminium disk). We add an accelerating mass of 10 g to the outer drive pulley.

Wrap the cord with the accelerating mass around the inner drive pulley ($r = 1.6 \text{ cm}$) until you reach the required height of fall. Hold the acrylic disk with a finger (there are reference values for a height of fall of $h = 0.8 \text{ m}$). You can see the current height (distance between holder for slotted weights and ultrasonic sensor) on the ULAB.

Please try to reduce possible interfering effects, as for instance vibrations or rotation of the holder for slotted weights, as much as possible before the start of the measurement.

Release the acrylic disk and simultaneously start the Coach data acquisition by clicking on "Start" in the menu bar.

The holder for slotted weights is accelerated and puts the acrylic disk into rotation. The ultrasonic sensor measures the signals 20 times per second and transmits them to the ULAB and the computer where they are pictured in a distance-time diagram and we receive a distance-time function.

Catch the falling mass before the cord is completely unwrapped. You can do that by either stopping the acrylic disk or by catching the slotted weight. For our evaluation we only deal with the accelerating mass.

ROTATION – MOMENT OF INERTIA MEK 3.3

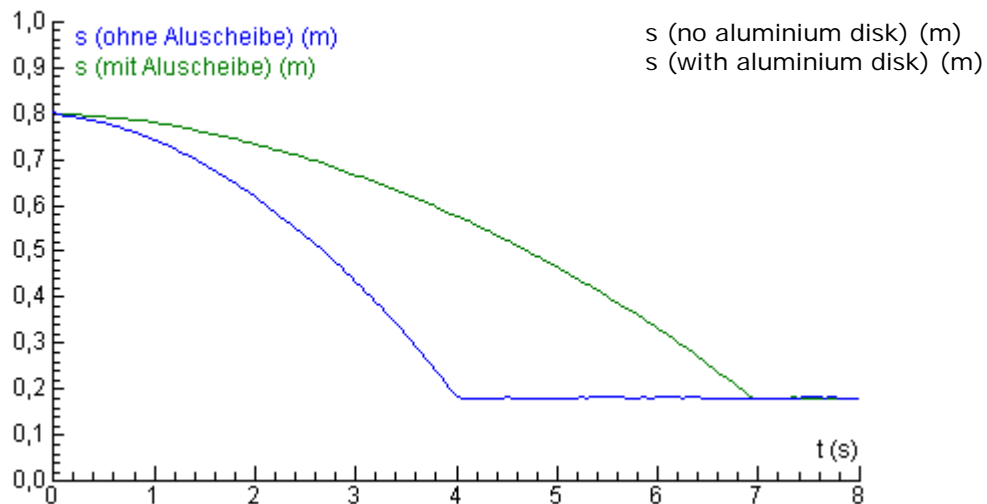


Figure 1: Distance-time diagrams for outer drive pulley and different moment of inertia

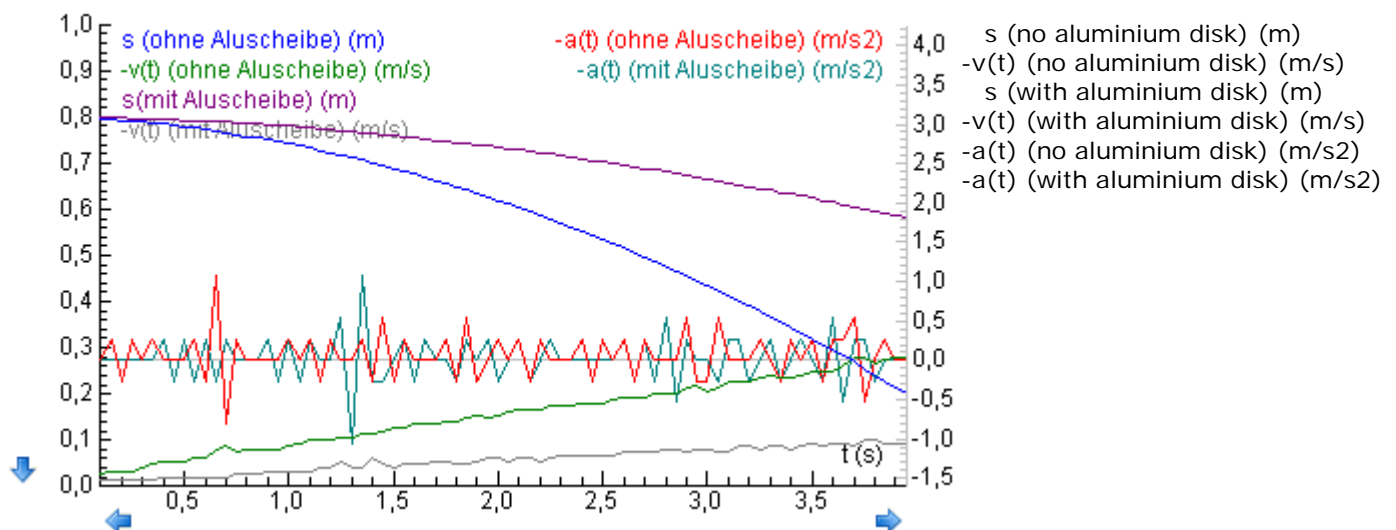


Figure 2: Diagram to determine the acceleration of both motions of fall

As in the next to last experiment, we determine the acceleration by fitting the data points with a quadratic function. The first coefficient is related to the half of the acceleration (the negative sign is insignificant), the second coefficient corresponds to the initial velocity and the third to the initial height. The acceleration, the accelerating mass and the radius can be substituted into equation (4) to calculate the acting torque. Fill in the value in the table below.

If you do not want to cancel the recorded function, you must save the data. Right-click in the diagram and choose "Copy Column" and "s". In that way the recorded data will be saved in another column and is not replaced by the new data when you carry out a new measurement.

ROTATION – MOMENT OF INERTIA

MEK 3.3

Experiment 1b:



Measuring time: 6 s

Double the accelerating masses by putting a second 10 g slotted weight on the holder for slotted weights and repeat the experiment. Evaluate the data like before and fill in the values in the table below.

	m_{total} [kg]	m_b [kg]	r [m]	$a/2$ [m/s ²]	a [m/s ²]	v [m/s]	s [m]	N [kgm ² /s ²]
No aluminium disk	0,135	0,01	0,016	0,0315	0,0630	0,0280	0,8007	0,001560
No aluminium disk	0,145	0,02	0,016	0,0663	0,1326	0,0411	0,7980	0,003097
With aluminium disk	0,337	0,01	0,016	0,0112	0,0224	0,0114	0,8011	0,001566
With aluminium disk	0,347	0,02	0,016	0,0227	0,0454	0,0160	0,8006	0,003125

Table 1: Torques acting on the outer drive pulley

Optional:

We can vary the torque by putting the aluminium disk onto the acrylic disk. Repeat the experiments by using accelerating masses of 10 g and 20 g. Fill in the values in the table above. There should not be a big difference between these torques and the torques calculated before.

Experiment 2:



Measuring time: 13 s

In the next experiment we are going to measure the torque acting on a rotating body (acrylic disk, also possible acrylic-aluminum disk). Now we put an accelerating mass of 40 g on the inner drive pulley of the shaft wheel.

Wrap the cord with the accelerating mass (holder of the slotted weights, one 20 g slotted weight and one 10 g slotted weight) around the inner drive pulley ($r = 0.4$ cm) of the shaft wheel until you reach the required height of fall. Hold the acrylic disk with a finger (there are reference values for a height of fall of $h = 0.8$ m). You can see the current height (distance between holder for slotted weights and ultrasonic sensor) on the ULAB.

Please try to reduce possible interfering effects, as for instance vibrations or rotation of the holder for slotted weights, as much as possible before the start of the measurement.

Release the acrylic disk and simultaneously start the Coach data acquisition by clicking on "Start" in the menu bar.

The evaluation of the data is similar to that in the experiments before. The experiment can be done with or without the aluminium disk. Fill in the values in the table on the next page.

ROTATION – MOMENT OF INERTIA

MEK 3.3

	m_{total} [kg]	m_b [kg]	r [m]	$a/2$ [m/s ²]	a [m/s ²]	v [m/s]	s [m]	N [kgm ² /s ²]
No aluminium disk	0,165	0,04	0,004	0,0104	0,0208	0,0270	0,8071	0,001566
With aluminium disk	0,367	0,04	0,004	0,0038	0,0076	0,0113	0,8087	0,001568

Table 2: Torques acting on the inner drive pulley

Please note:

In the experiment 1b the accelerating mass was twice as big as in experiment 1a. Therefore the acceleration and the torque are also doubled.

An increase of the moment of inertia has no consequences on the torque.

In the second experiment the accelerating mass was four times bigger than the mass in experiment 1a. We expected that also the acceleration and the torque would be four times bigger. That was compensated by reducing the radius to a quarter of the radius in experiment 1a.



Conclusion:

The torque is directly proportional to the acting force and the distance between the center of rotation and the application point of the force. The acting force is directly proportional to the accelerating mass and the resultant acceleration (approximately).



Student Experiments

© Fruhmann GmbH
NTL Manufacturer & Wholesaler

Werner von Siemensstraße 1
A - 7343 Neutal
Austria

www.ntl.at